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PROPAGATION OF A SHOCK WAVE IN A LOCALLY ANISOTROPIC MEDIUM CONTAINING CHANNELS WITH ELEVATED THERMAL DIFFUSIVITY

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We consider the interaction between a shock wave and thin extended channels with elevated a thermal diffusivity. It is shown that a thermal perturbation ahead of the shock wave front leads to the effect of a "warm layer," i.e., to the formation of a large-scale self-similarly growing precursor.

When a shock wave (SW) moves in a medium containing thin extended channels of reduced density oriented at a right angle or some other not very small angle to the shock front, distortion of the wave front occurs, and ahead of it a wedge-shaped or cone-shaped perturbation (precursor) arises. At the present time, this regime has been fairly thoroughly studied for channels in the form of plane layers or cylinders (see, e.g., [1–4], where it is shown that at rather large times, when an SW traverses a distance in a channel greatly exceeding its thickness, the flow in the precursor has a self-similar nature).

Let us consider another form of the local anisotropy of a medium, namely, a thin extended channel with elevated thermal diffusivity produced, for example, by means of an oriented addition of a certain admixture with atomic number Z lower than that of the environment. If the concentration of the admixture is low, then all the thermodynamic properties of the admixture-containing layer will be the same as in the "uncontaminated" medium, but, because of the strong dependence of the optical properties of a substance and other characteristics of it on the atomic number, the coefficients of radiative and electronic heat conduction may differ appreciably from the corresponding background values. A similar situation can also be encountered when a perturbing layer consists entirely of another substance but both gases are preliminarily heated up to rather high temperatures. At high temperatures and a high degree of ionization of various gases their "apparent" atomic weights will almost coincide [5]. Correspondingly, at the same initial pressure and initial temperature the initial densities and heat capacities of gases will also be nearly equal. At the same time the values of the coefficients of radiative heat conduction depend on the mean charge $\bar{Z}$ in an ionized gas (or on the atomic number Z in the case of complete ionization) more strongly than the equation of state. Therefore, there are situations where despite close densities of the substances of the layer and the main gas their thermal diffusivity coefficients may, generally speaking, differ by orders of magnitude.

Suppose that an SW propagates through such a locally anisotropic medium white heating the gas up to substantially higher temperatures than the initial temperature. Since the rate of heat transfer along the channel is rather high, a thermal precursor arising ahead of the SW extends in the direction of the channel, forming a zone with a temperature higher than that of the surrounding medium. As shown by results of numerical calculations [6] at the two-dimensional nonstationary problem of propagation of a pure heat wave (HW) in a medium containing channels with elevated thermal diffusivity, in the limit the motion of the thermal precursor occurs not self-similarly, but in a steady-state fashion, since over time a balance is attained between the heat fluxes: that entering the precursor and that leaving the precursor through its side surface. The length of the steady-state thermal precursor is directly proportional to the channel thickness and to the ratio of the thermal diffusivity coefficients in the channel and the surrounding medium. Therefore, by increasing the thickness of the channel or changing the composition

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of the admixture (thus increasing the thermal diffusivity in the channel), it is possible to attain a length of the thermal precursor such that the time for the SW front to cover this distance will be sufficient for the development of gasdynamic processes in the direction perpendicular to the channel.

As a result of transverse expansion of the channel, ahead of the SW front a new channel of limited length l arises that is continuously generated by the mechanism of heat conduction has a density ρ lower than the density ρ_0 of the surrounding medium. The presence of this channel will lead, in turn, to the formation of a gasdynamic precursor ahead of the SW front (similar to that observed in experiments and calculations in [1–4]). The parameters of this precursor will be determined by the dynamics of the lateral expansion of the initial channel responsible for the selection of the characteristic "working" density ρ_c in the channel.

The mass of the gas in the forming channel of lower density turns out to be much greater than the initial mass in the channel with elevated thermal diffusivity, with the thickness of the expanded channel also being much larger. In essence, the role of the channel of elevated thermal conductivity comes down to the heating of gas along the channel axis up to a temperature close to that of the SW front and to the initiation of the lateral expansion of gas into the surrounding medium. The parameters of the initial channel (thickness and thermal diffusivity) determine only the length l of the plug of heated gas and, correspondingly, the time t_c of its expansion up to the density $\rho_c < \rho_0$ during which the shock front covers the distance l.

We consider the above-described thermal mechanism underlying the initiation of shock front perturbation using as an example the following model problem. Suppose that ahead of the front of a stationary plane SW generated by a piston and propagating with the velocity D_s a channel of finite length l with temperature equal to the temperature T_s behind the SW front always exists. Such a situation arises, for example, when a strong intensely radiating SW (supercritical in the terminology of [5]) moves through a gas, when the effects of radiative heat conduction afford the heating of the gas up to the postfrontal temperatures at a finite length ahead of the shock front, i.e., a stationary thermal "tongue." However, we will not specify the mechanism of nonlinear heat conduction, but rather consider the model case of a power-law dependence of the thermal conductivity coefficient on temperature and density:

$$\lambda = C_0 T^3/k, \tag{1}$$

where the coefficient $k(T, \rho) = C_1 T^{-\alpha_1}/\rho^{\beta_1}$ (in the case of radiative heat conduction $C_0 = (16/3)\sigma$, where σ is the Stefan–Boltzmann constant and k has the meaning of the mean Rosseland absorption coefficient). We also consider the specific internal energy of the gas to be a power function of temperature and density:

$$e = C_2 T^{\alpha_2} \rho^{-\beta_2}, \tag{2}$$

while the effective specific heat ratio

$$\gamma = 1 + p/(\rho e) \tag{3}$$

is assumed to be constant.

Since the influence of the elevated thermal diffusivity in the channel comes down to just the effective heating of the gas in it over a length l exceeding the length of heating in the main gas, while the initial thickness and density of the channel "are forgotten" in the process of its lateral expansion, then, to simplify the analysis and make the results more universal, we will consider the limiting case where the initial thickness of the channel is equal to zero. In the two-dimensional plane geometry of flow, when the initially unperturbed shock front is parallel to the Oy axis, this case is realized via an additional boundary condition on the symmetry axis (or rigid wall) Ox:

$$T = T_s \quad \text{for} \quad x_s \le x \le x_s + l, \tag{4}$$

where $x_s(t)$ is the coordinate of the perturbed SW front in a channel of zero initial thickness. We note that to perform a numerical calculation of a two-dimensional nonstationary problem in a real situation (without the simplifying condition (4)), where the initial thickness of the channel increases by orders of magnitude in the process of its expansion but nevertheless remains small in comparison with the dimensions of the gasdynamic precursor, which increases, with time (even upon expansion the channel remains "thin"), it is necessary to use different-scale movable difference grids with a large number of nodes; such calculations demand very fast computers.

First, we consider the dynamics of the lateral expansion of gas in a channel and estimate the values of its characteristic parameters. If at the time of arrival of an SW at the section of an expanded channel its thickness is smaller than l, the process of expansion can be considered one-dimensional (plane). The value of the velocity of the main SW D_s, and together with it of the temperature T_s of the SW front and the channel, is assumed to be rather large in order to neglect the effects of the back pressure of the surrounding gas, which is considered to be absolutely cold. Therefore, it is necessary to determine the parameters of the motion and heating of initially quiescent absolutely cold gas with the initial density ρ_0 in the half-space $y \geq 0$ with account for the boundary conditions: $T(0, t) = T_s = \text{const}$, $v(0, t) = 0$.

The qualitative picture of the process is as follows. The effects of heat conduction cause the propagation of an HW through the gas from the hot "wall" ($y = 0$). At the beginning this wave is fast (its velocity is $D_T \sim t^{-0.5}$), the heated gas behind its front has no time to go into motion, and its density ρ hardly changes compared with the initial density ρ_0 (the relative density of the gas is $\omega = \rho/\rho_0 \approx 1$). With the passage of time the velocity of the HW drops, and the high pressure of the heated gas sets it in motion. An SW begins to form, which overtakes the HW at the time $t = t_*$. In the gas a nonstationary complex arises that consists of the HW and the SW, which propagate in space with virtually the same velocities, which decay weakly in time. Because of the high velocities of sound in the hot region behind the HW front, the pressure there turns out to be equalized at each moment of time; simultaneously the high thermal conductivity ensures the isothermality of the flow, and therefore the density in this regions is also virtually equalized, with $\omega < 1$. The gas temperature behind the HW front (which plays the part of a retarding piston) is much higher than the temperature behind the SW front, the mass flow rate through the HW front is extremely small, and its velocity $D_T(t)$ changes in a narrow interval from the velocity $D_1(t)$ of the SW at $t \approx t_*$ to the minimum value $2D_1(t)/(\gamma + 1)$ at $t >> t_*$ (the "piston" turns out to be not rigid, but permeable, as though porous, with the "pores" closing with time). In this connection, the gas pressures behind the HW front and in the region between the HW and SW fronts practically coincide. Thus, the magnitude of the pressure turns out to be constant over the entire space from the wall to the SW front, decreasing with time in response to the decrease in the heat flux from the wall and the gas expansion. An overwhelming portion of the mass of the gas captured by an SW is raked up into a relatively cold, heavy, and thin "crust" between the HW and SW fronts. The mass of the heated and rarefied gas behind the HW front is much smaller than the mass of the crust, but it is precisely in it (and not in the crust) that the main portion of the total energy entering the system from the hot wall is contained in the form of internal energy.

Similar stationary flow regimes with an HW–SW complex under conditions of nearly equalized pressure have been studied comprehensively in the theory of classical combustion (the modes of combustion from the closed end of a tube [7]), and they are also realized during the "burning" of a substance in a laser beam (the mode of "ejection" [8]).

The condition of pressure equalization and the consequence of energy balance in the system can be written in the form

$$p/(\gamma - 1) = \omega\rho_0 e_2 = (\gamma + 1)/(\gamma - 1)\,\rho_0 e_1\,, \tag{5}$$

$$\omega\rho_0 D e_2 = q_2 = (-\lambda \partial T/\partial y)_2\,. \tag{6}$$

Here the subscripts 1 and 2 denote the values of parameters behind the SW front and on the wall, respectively. It is assumed that $D_1 = D_T = D$. In (6) the kinetic energy of the gas behind the HW front and the total energy of the crust are neglected. Since the SW is strong, $e_1 = 2/(\gamma + 1)^2 D^2$. The value of $e_2(T_s, \omega\rho_0)$ is determined from (2). Conditions (5) and (6) make it possible to find power-law relationships for the change in the characteristic parameters ω, p, D with time. In order to avoid large errors in the determination of dimensional constants in front of the exponents, it is necessary that the heat flux q_2 from the hot wall be approximated accurately in (6), since

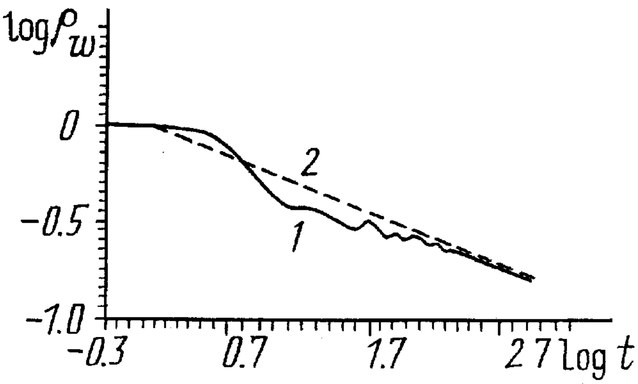

Fig. 1. One-dimensional flow in an expanding channel. Density of gas ρ_w near the heated "wall" vs time: 1) calculation; 2) estimate (9).

both the temperature and the density change sharply simultaneously in the vicinity of the HW front. Since the pressure along the HW front is continuous, it is better that the heat conduction coefficient λ be approximated in the variables T and p. In this case, the temperature dependence of q_2 is approximated in the form

$$q_2 \sim (T^{\alpha_3+3}\, \partial T/\partial y)_2 \sim (\alpha_3+4)^{-1}\, (\partial\, (T^{\alpha_3+4}/\partial y)_2 \approx (\alpha_3+4)^{-1}\, T_s^{\alpha_3+4}/(Dt)\,, \tag{7}$$

where $\alpha_3 = \alpha_1 + \beta_1\alpha_2/(1-\beta_2)$. The presence of the factor $(\alpha_3+4)^{-1}$ in approximation (7) (much less than unity in actual cases) increases significantly the accuracy of the estimation of parameters.

As a result, we obtain the following power-law relations for the characteristic parameters:

$$\omega = (t_*/t)^{1/(2+\beta_1-2\beta_2)}\,, \quad p = p_*\, (t_*/t)^{(1-\beta_2)/(2+\beta_1-2\beta_2)}\,,$$

$$D = D_*\, (t_*/t)^{0.5(1-\beta_2)/(2+\beta_1-2\beta_2)} \tag{8}$$

for $t \ge t_* = 2C_0T_s^{4+\alpha_1-2\alpha_2}/(C_1C_2^2(4+\alpha_1+\beta_1\alpha_2/(1-\beta_2))(\gamma^2-1)\rho_0^{1+\beta_1-2\beta_2})$, where $p_* = C_2(\gamma-1)T_s^{\alpha_2}\rho_0^{1-\beta_2}$, $D_* = (0.5C_2(\gamma^2-1)T_s^{\alpha_2}\rho_0^{-\beta_2})^{0.5}$.

The above estimate makes it possible to follow the change in the characteristic parameters in a rarefied channel behind the HW front up to the time $t = t_*$ when $\omega = 1$. In this case, the quantity t_* has the meaning of the time at which the shock wave overtakes the thermal wave, while p_* and D_* are the characteristic pressure and velocity at the moment of overtaking. Strictly speaking, estimate (8) holds for times $t >> t_*$, when sonic perturbations have time to propagate multiply throughout the channel volume, and the mode of heating and expansion of gas with equalized pressure is established. However, we may extend this estimate approximately up to the time $t = t_*$, the more so that the very concept of overtaking is purely conventional, since in reality gasdynamic and thermal processes proceed simultaneously. Thus, we may assume that $\omega \equiv 1$ when $0 < t < t_*$, and it can be found from Eq. (8) when $t \ge t_*$.

We consider a specific example. Suppose $\alpha_1 = 0$, $\beta_1 = 1.64$, $\alpha_2 = 1.5$, $\beta_2 = 0.12$, and $\gamma = 1.24$. These values of the exponents in laws (1), (2) and of the effective specific heat ratio roughly approximate the thermodynamics of an air plasma [5] and the mean Rosseland coefficient of radiation absorption in it (the latter coefficient in the region of temperatures $T < 10$ eV). Here, we do not cite the values of the dimensional constants C_0, C_1, C_2, or of the parameters T_s and ρ_0, since they are used to make quantities dimensionless in the numerical calculations whose results will be given below. In this case, the time is normalized by the quantity t_*, velocities by D_*, distances by D_*t_*, pressures by p_*, temperatures by T_*, and densities by ρ_0. From now on, we will use only dimensionless parameters without additional notation.

In the case considered, Eqs. (8) yield

$$\rho \equiv \omega = t^{-0.294}\,, \quad p = t^{-0.259}\,, \quad D = t^{-0.13}\,. \tag{9}$$

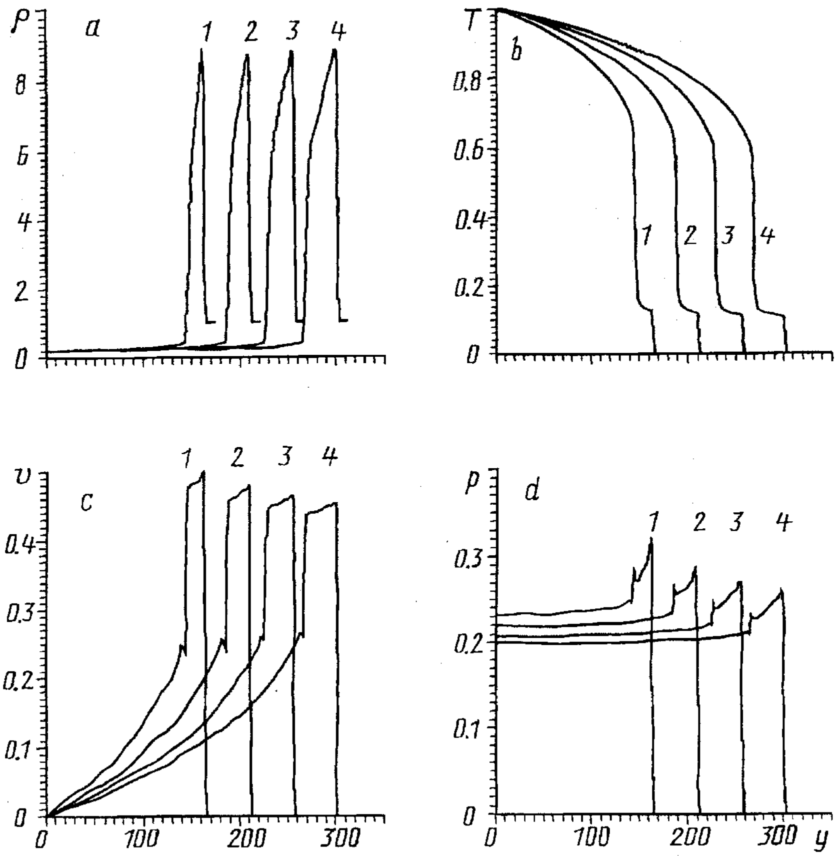

Fig. 2. One-dimensional flow in an expanding channel. Distribution of the densities ρ (a), temperatures T (b), velocities v (c), and pressures p (d) over the height y from the heated wall at the moments of time t = 260 (1), 346 (2), 432 (3), and 518 (4).

We compare the results of estimate (9) for the characteristic value ω of the relative density in a channel with the value $\rho_w \equiv \rho(0, t)$ obtained in a numerical calculation of a similar one-dimensional nonstationary problem with the same set of determining parameters (see Fig. 1). The figure demonstrates rather satisfactory agreement between the estimate and the calculation even in the region of establishment of a regime at times $t \sim 1$.

We no let us estimate the working density $\omega_c(t_c)$ in an expanded channel arising ahead of an SW in a two-dimensional flow, having equated the time of expansion t_c to the time l/D_s of propagation of the SW along the channel length. In the two-dimensional problem analyzed $D_s \approx 2.67$, $l \approx 57.6$, and hence the characteristic time of channel expansion is $t_c \approx 21.6$, and from Eqs. (9) we obtain that the density in a channel section at the time of arrival of the SW there is equal to $\omega_c \approx 0.405$. We note that the relatively small exponent in the function $\omega(t)$ from Eqs. (8) or (9) produces conditions for stable selection of the determining parameter ω_c in a two-dimensional flow.

According to Eqs. (9), the characteristic velocity of the channel boundary at the time t_c of termination of expansion is $D_c \approx 0.671$, while its characteristic thickness is $y_c \approx 16.5 < l$, and therefore the use of a one-dimensional channel expansion model for estimation seems to be justified. The channel thickness $y_* = 1$ at the time $t = 1$ of the overtaking of the heat wave by the shock wave is much smaller than the limiting thickness of the channel y_c; we note that the value of y_* coincides in order of magnitude with the characteristic length of the heating tongue ahead of the front of the main SW in a two-dimensional flow. The characteristic pressure in the channel at this very moment, $p_c \approx 0.451$, is 16 times smaller than the pressure, $p_s \approx 7.14$, behind the front of the unperturbed SW incident on the channel end face, and the effects of the backpressure are insignificant; as demonstrated by the two-dimensional calculation whose results are given below, in the case of formation of a gasdynamic precursor too the pressure behind a normal shock propagating in the channel is about an order of magnitude higher at the precursor apex than the pressure p_c in the channel; i.e., under the conditions considered the characteristics of a

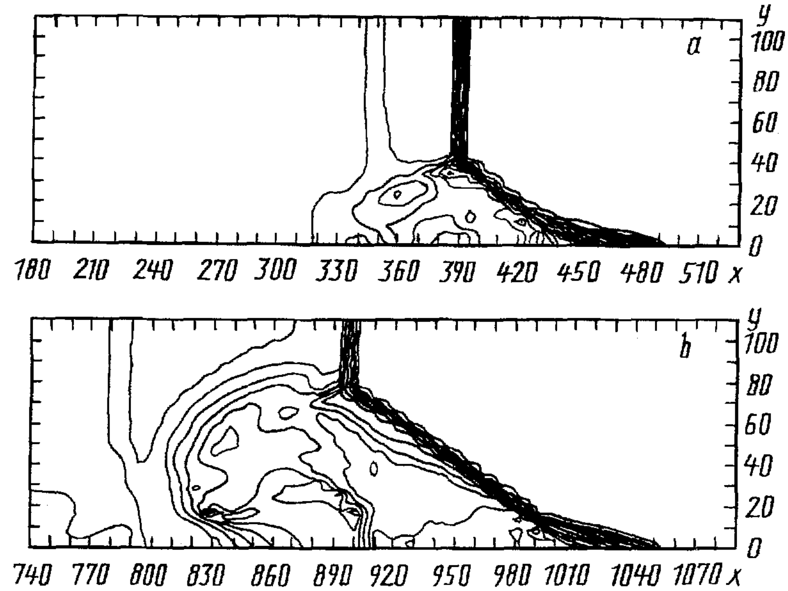

Fig. 3. Temperature field in a two-dimensional flow at the times $t = 140$ (a) and 320 (b).

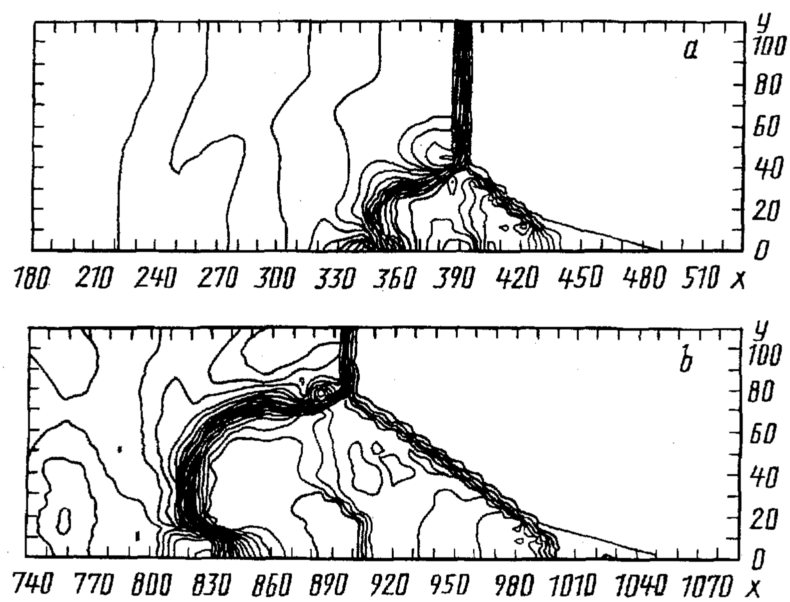

Fig. 4. Field of pressures in a two-dimensional flow at the times $t = 140$ (a) and 320 (b).

two-dimensional flow are determined by just one parameter, namely, the magnitude of the relative rarefaction in a channel ω_c.

In Fig. 2 distributions of the parameters in a channel over the distance from the wall are given that were obtained in a one-dimensional calculation. They illustrate the qualitative picture of the dynamics of channel expansion presented above. Naturally, a real flow turns out to be somewhat more complex than that in the simple model suggested for estimation, with the difference being especially conspicuous in the vicinity of the HW front and in the interval between the HW and SW fronts. Nevertheless it is evident that the condition of equalization of the pressure in an expanding channel, which underlies the estimation, is fulfilled with good accuracy, thus ensuring reasonable agreement between the calculated and estimated values of rarefaction in a channel.

Figures 3 and 4 illustrate the results of a two-dimensional numerical calculation. The formation of a large-scale gasdynamic precursor ahead of the main shock front and the self-similar character of its propagation are clearly seen. In contrast to the results of earlier calculations [4] of a similar problem in a purely gasdynamic formulation without account for the effects of heat conduction (with an extended channel of finite thickness with a given constant rarefaction ω_0), the self-similar precursor arising in the present case joins, at the apex, with the stationary wedge of the thermal heating tongue formed upon expansion of a channel of "zero" thickness that is induced by heat conduction. In the calculation the divergence angle of the thermal wedge is equal to $\varphi_T \approx$ arctan (0.28), which is in good agreement with the estimated value of this parameter $\varphi_T \approx$ arctan $(y_c/l) \approx$ arctan (0.29). The wedge angle of the gasdynamic precursor in the calculation is equal to $\varphi \approx$ arctan(0.82) $> \varphi_T$. The value of φ can be used for determining the characteristic density ω_c in a channel in accordance with a simple estimate given in [2–4]:

$$\sin\varphi = \sqrt{\omega}\,, \tag{10}$$

whose accuracy for strong SWs is quite high. According to Eq. (10), we have $\omega_c \approx 0.4$, which practically coincides with the above estimate of this parameter (the same value of the effective density ω_c was obtained directly in the two-dimensional calculation). The remaining qualitative picture of the developing two-dimensional flow does not undergo substantial changes compared to the purely gasdynamic case. The velocity of propagation of the apex of the shock-thermal wedge of the precursor along the channel axis is $D_p \approx 2.87$ (a universal constant of the beginning of stagnation), and the relative velocity of its growth $\zeta = (D_p - D_s)/D_s \approx 0.075$ agrees satisfactorily with the results of gasdynamic calculations of the parameter $\zeta(\omega)$ for strong SWs [4].

The foregoing results show that the propagation of an SW in a medium with local nonuniformities of the type of thin extended channels of elevated thermal diffusivity leads to the effect of a "warm layer," which is a large-scale deformation of a shock front interacting with a thin rarefied channel.

NOTATION

T, temperature; p, pressure; ρ, density; v, velocity; e, specific internal energy; λ, thermal conductivity coefficient; γ, effective specific heat ratio; q, heat flux density; t, time; x, y, Cartesian coordinates.

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